Compute the integrals

$$\int \sin^3 x \sec^2 x \, dx$$

$$\int \sin x \cos(2x) \, dx$$

$$\int \sin(2x) \cos(3x) \, dx$$

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$$\int \sin^3 x \sec^2 x \, dx$$

$$= \int \sin x \left(1 - \cos^2 x\right) \frac{1}{\cos^2 x} \, dx$$

$u = \cos x \Rightarrow du = -\sin x \, dx$

$$\to = -\int (1 - u^2) \frac{1}{u^2} \, du$$

$$= -\int \frac{1}{u^2} - 1 \, du$$

$$= -\left(-\frac{1}{u} - u + C_1\right)$$

$$= u + \frac{1}{u} + C$$

$$= \cos x + \frac{1}{\cos x} + C$$

$$\int \sin x \cos(2x) \, dx$$

$$= \int \sin x \left(\cos^2 x - \sin^2 x\right) dx$$

$u = \cos x \Rightarrow du = -\sin x \, dx$

$$\to = -\int u^2 - (1 - u^2) \, du$$

$$= -\int 2u^2 - 1 \, du$$

$$= -\left(\frac{2u^3}{3} - u + C_1\right)$$

$$= -\frac{2u^3}{3} + u + C$$

$$= -\frac{2\cos^3 x}{3} + \cos x + C$$

$\sin 2x$
$= 2\sin x\cos x$

$\int \sin(2x)\cos(3x)\,dx$

Let $u = \cos x$.
$\Rightarrow du = -\sin x\,dx$

$= \int \sin 2x(\cos 2x\cos x - \sin 2x\sin x)\,dx$

$= \int (\cos 2x\cos x - 2\sin^2 x\cos x)\sin 2x\,dx$

$= \int (\cos 2x\cos x - 2(1-\cos^2 x)\cos x)\sin 2x\,dx$

$= \int (\cos 2x\cos x - 2\cos x + 2\cos^3 x)\sin 2x\,dx$

$= \int ((2\cos^2 x - 1)\cos x - 2\cos x + 2\cos^3 x))\,2\sin x\cos x\,dx$

$= -2\int ((2u^2-1)u - 2u + 2u^3))\,u\,du$

$= -2\int 2u^4 - u^2 - 2u^2 + 2u^4\,du$

$= -2\int 4u^4 - 3u^2\,du$

$= -2\left(\frac{4u^5}{5} - \frac{3u^3}{3} + c_1\right)$

$= -\frac{8}{5}u^5 + 2u^3 + C$

$= -\frac{8}{5}\cos^5 x + 2\cos^3 x + C$